ANALYSIS AND SYNTHESIS OF WAVEGUIDE MULTI-APERTURE DIRECTIONAL COUPLERS

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Introduction. The conventional treatment of directional couplers formed by two parallel waveguides coupled by a number of equally spaced discrete apertures is based on a loose coupling theory which, for co-directional couplers, assumes that the individual coupled voltages of the apertures add in the forward direction and tend to cancel in the backward direction.[1] Assuming no multiple coupling or interaction effects, a first order expression for the isolation as a function of frequency and of the voltage amplitude couplings of the individual apertures is readily derived. These amplitudes may be tapered to give either Butterworth or Chebyshev performance and simple formulae for the amplitudes derived.[1,2] In the case of fairly tightly coupled waveguides, e.g. 3 dB, it is preferable to use a set of superimposed arrays,[1] which results in a shorter coupler since the majority of the holes in this design method are of equal diameter.

When the overall coupling of the directional coupler becomes strong, then the theory outlined above tends to become inaccurate. For example, if the number of apertures is increased, the assumption of simple addition of the voltage amplitudes to give the total amplitude coupling must break down well before a coupling of unity is approached. It is then preferable to use the accurate formula for the voltage coupling k given by[3]

$$k = \sin \left[\sum_{i=1}^{n} \sin^{-1} k_i \right] \qquad (1)$$

where k_i is the voltage coupling of the i^{th} aperture. The question now arises as to what happens to the isolation under this strong coupling condition. This may be answered only by developing an exact or more exact theory than that outlined above. The basis for such a theory has existed for a considerable time,[3] but apparently has not been applied previously to the multi-aperture coupler problem.

New method of analysis. In the new treatment a rigorous circuit approach to the problem is adopted, and only the possibility of direct interaction effects (i.e. involving evanescent modes or fringing fields) between adjacent apertures is neglected. It is known that a small aperture behaves as a lumped reactance in series or shunt with the two waveguides. An aperture in a common narrow wall behaves as a shunt inductance, as does a narrow longitudinal slot in a broad-wall coupler. A transverse slot in the latter gives series inductive coupling, and a circular aperture gives a combination of inductive and shunt capacitive coupling.

The Bethe small aperture theory is employed, and the effects of large aperture dimensions are included by replacing the simple reactances by resonant elements.[4] Finite common wall thickness, which attenuates the coupled waves, adds another readily calculable frequency-dependent term.[4] The directional coupler is equivalent therefore to two dispersive transmission lines coupled at discrete intervals by means of reactive elements in series and/or shunt, having a prescribed frequency dependence.

This 4-port equivalent circuit may be analyzed exactly, and in the most commonly encountered case of identical coupled waveguides, the analysis utilizes the well-known concept of even-and-odd-mode two-port equivalent networks.[1] The simplest examples are for apertures having a simple equivalent circuit, e.g. narrow-wall multi-hole couplers having discrete shunt inductive coupling (apart from the hole resonance and wall thickness effects). The equivalent circuit is shown in Fig. 1, which also indicates the symmetry plane for the normal mode analysis. The odd-mode two-port circuit is a cascade of shunt inductances separated by lengths of transmission lines, while the even-mode two-port is simply a length of uniform transmission line.

Analysis of previous designs. The analysis method has been used to investigate the performance of many couplers designed on the basis of the published theories. The directivity results for X-band WR90 3 dB narrow-wall 18-hole couplers centered at 9.1 GHz are shown in Fig. 2. The conventional Chebyshev design was for the band 8.70-9.55 GHz, but the actual performance is nearer to maximally-flat than to equal-ripple. Considering the approximations made in the design theory the directivity is surprisingly good over a fairly wide band. A superimposed array consisting of five 6-hole binomial arrays was then designed, leading to individual voltage couplings for the first 9 holes of the symmetrical 18-hole array of[1]

1 5 10 11 10 11 11 10 11

The directivity is better than 60 dB over the band 8.3-10.0 GHz, but is of rather narrower bandwidth to the 40 dB points than the previous example. In general therefore, the design techniques now in use give good results over quite broad bandwidths, but not in agreement with the simple theory on which they are based.

New design. A more exact synthesis method is now proposed which gives results very close to the predictions. In the case of the simple shunt inductive type of coupling shown in Fig. 1 the odd-mode circuit is designed using a distributed low-pass prototype filter synthesized exactly.[2,5] Since the even-mode circuit is matched, i.e. $\Gamma_e=0$, the voltage reflection coefficient and isolation are given by $\Gamma_o/2$. It has been found that if the bandwidth of the prototype is chosen to be that of the coupler, then the isolation is usually practically infinite, but the shunt susceptances and hence the aperture dimensions vary considerably (although less so than in the simple Chebyshev theory). It is preferable to design for a set isolation having a bandwidth much greater than that strictly required. The only parameter remaining is R, the product of the prototype junction VSWR's, which determines the coupling. It can be shown that for a given voltage coupling k,R is given very accurately by the expression

$$R = \left[\frac{1+\sin(\frac{2}{n}\sin^{-1}k)}{1-\sin(\frac{2}{n}\sin^{-1}k)}\right]^n \qquad (2)$$

n being the number of apertures. the odd-mode circuit is formed by choosing the susceptance values to give a mid-band VSWR equal to that of the corresponding junction VSWR's of the prototype. On the basis of this theory the spacing between apertures should be chosen to give the well-known synchronous condition, e.g. the first value of electrical length in Fig. 1 is given by

$$\theta_1 = \pi - \tfrac{1}{2}\left(\tan^{-1}\frac{2}{|b_1|} + \tan^{-1}\frac{2}{|b_2|}\right) \qquad (3)$$

which is slightly greater than $\pi/2$. However it will be shown that equal hole spacing gives equally satisfactory results.

The 18-hole 3 dB coupler was re-designed using the new theory to give 92 dB isolation, i.e. $\Gamma_0/2 = 0.000025$, giving a prototype filter VSWR of 1.0001. The existing tables[5] do not include prototypes of such good VSWR, and new sets have been specially computed for this work. The directivity computation of the new design in Fig. 2 shows an approximate equal-ripple directivity at the predicted level. The deviation from the prototype is due to the frequency sensitivity of the aperture susceptances, which causes bandwidth contraction and suppression of ripples at the band edges. The spacing between holes in this design correspond to the synchronous condition [3], but equal hole spacing gives excellent results, as shown also in Fig. 2.

It is possible to apply the new technique without access to exact distributed low-pass filter prototypes by using an approximate derivation of the latter.[2] The equivalent of the old superimposed array method, where most of the holes are equal in diameter, has been obtained by such means, and satisfactory results obtained.

Experimental results for the 18-hole coupler with equal hole spacing show a directivity of > 43 dB over the band 8.7-10.5 GHz. Naturally the effects of tolerances, of undesired interactions, and experimental errors make it practically impossible to achieve a directivity of 80 dB or so, but the new theory gives a better and more reliable prediction of the performance.

Broad-wall couplers. At present more careful and extensive measurements have been conducted with broad-wall couplers, as described below. The equivalent circuit for each aperture in this case consists of a series inductor, a shunt inductor, and a shunt capacitor. The existence of series reactances means that the odd-mode network is non-trivial. The only unknown factor in the case of circular apertures is the determination of the effective common-wall thickness. It has been found that this is a function both of the physical wall thickness and of the hole diameter, and may be represented very well by a linear expression of the form:-

Effective thickness = const. x hole diameter + actual thickness

Results for single aperture couplers will be presented to show the close agreement between theory and experiment which has been obtained. This is indicated here by the results shown in Fig. 3 for an X-band coupler consisting of 6 sets of double holes. The coupling is predicted 0.4 dB too loose, but this is a result only of the simplified formula for attenuation caused by hole thickness, and may be corrected. The important point is that the coupling frequency variation is predicted accurately, and the directivity is also well reproduced in practice. Similar results have been obtained in several other cases.

The synthesis technique for broad-wall couplers is similar to that described for the narrow-wall case, but now we have distinct even and odd mode circuits. The technique used is to equate the mid-band amplitude coupling of each aperture to the reflection coefficient of corresponding steps of the distributed stepped-impedance prototype filter. Very short couplers having high directivity over entire waveguide bands have been designed in this way, and results will be presented.

References

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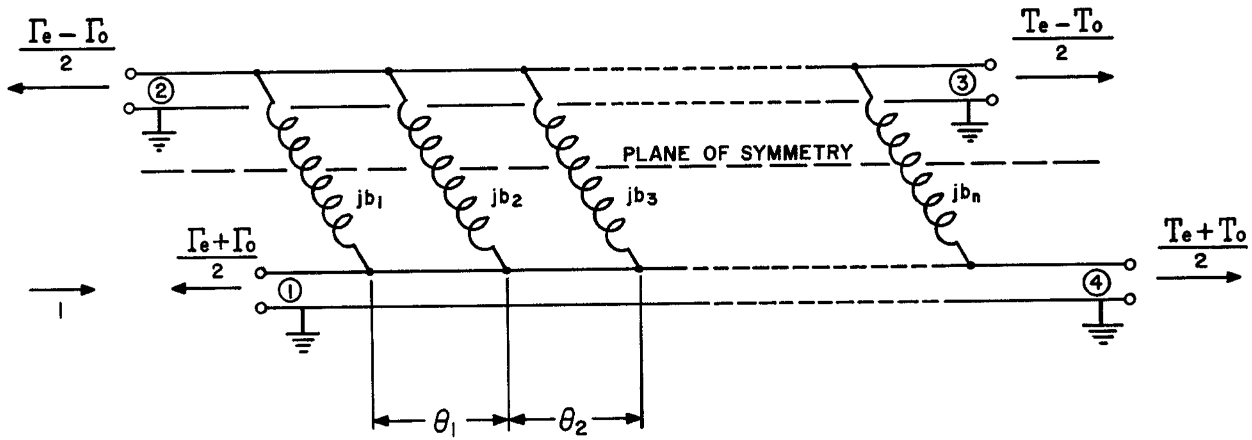
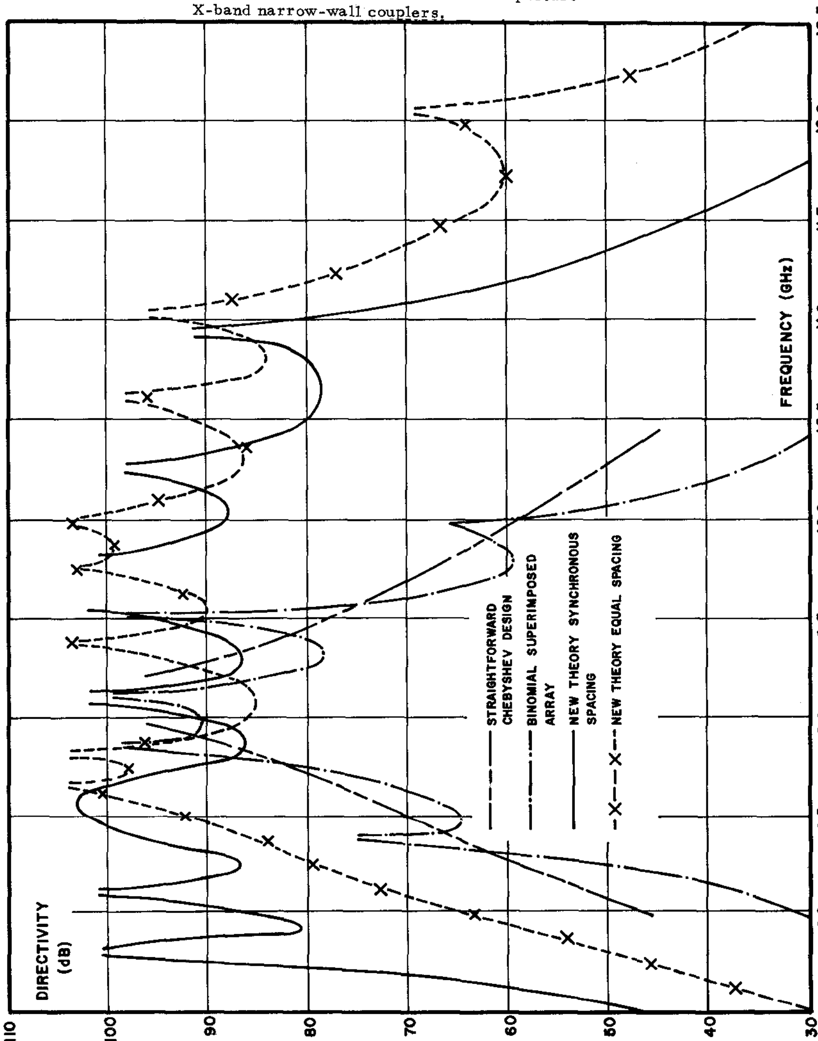


Fig. 1 Four-port equivalent circuit of an n-aperture narrow-wall directional coupler.

Fig. 2 Analyzed performance of several 18-aperture X-band narrow-wall couplers.



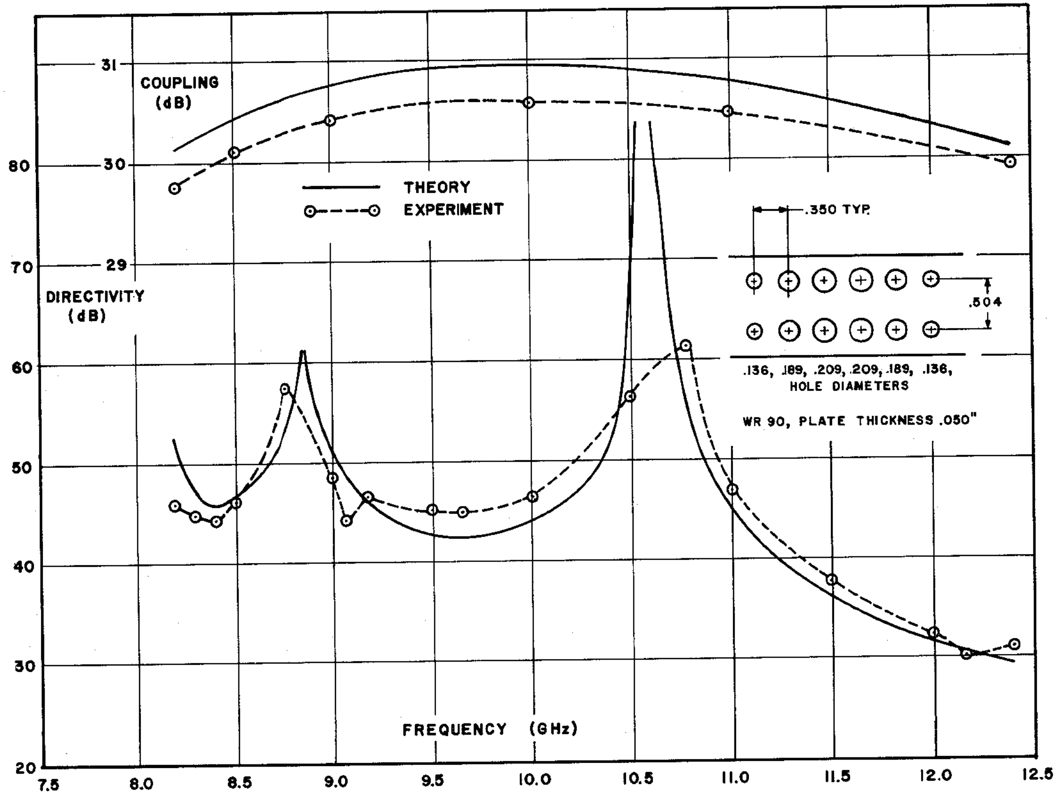


Fig. 3 Coupling and directivity of a 6-aperture X-band broad-wall coupler.